BLACK-SCHOLES AND BEYOND

OPTION PRICING MODELS

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by the delta of the option. The conclusion to all of this is:

> *The theoretical value of a vanilla European call option is completely determined by its delta, the risk-free rate of interest and the time to expiration.*

Summary

Let's review the main points covered so far. First of all, we have shown that the Black-Scholes formula gives rise to a hedging strategy for the short position of a vanilla European call. To show that the value of the hedging portfolio is equal to that of the option at every time, we need to: 1) know the delta of the option at every time, and 2) use the delta to determine the correct value of B_t at every time t. There is only one thing left to do: give formulas for Δ_t and B_t.

4.8 THE BLACK-SCHOLES FORMULAS FOR Δ_t AND B_t

Now that we understand the general idea of the Black-Scholes formula, let's actually see what it is. All we have to do is to give the formulas for Δ_t and B_t.

The formulas are given in terms of the cumulative normal distribution function, discussed in Chapter 2. They are:

$$\Delta_t = N(d_1), \quad d_1 = \frac{\log(S_t/K) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{4.8.1}$$

$$B_t = N(d_2)K, \quad d_2 = \frac{\log(S_t/K) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{4.8.2}$$

where

S_t = price of stock per share at time t
K = strike price
r = risk-free rate of interest
σ = volatility of stock under geometric Brownian motion model
$T - t$ = time until expiration
$N(\cdot)$ = cumulative normal distribution function

Combining this with equation (4.6.1), we present the Black-Scholes formula for vanilla European call options on a non-dividend-paying stock:

$$C_t = N(d_1) \cdot S_t - e^{-r(T-t)}K \cdot N(d_2). \tag{4.8.3}$$